A planar harmonic mapping $U = (u^1, u^2)$ on the unit disk $B \subset \mathbb{R}^2$ is simply a pair of harmonic functions on B. The theme of the talk is establishing conditions under which U is a global homeomorphism.

Given a homeomorphism Φ of ∂B onto a simple closed Jordan curve γ, set D to be the simply connected bounded open set determined by γ. A classical result of H. Kneser (1926) establishes that, if D is convex, the harmonic extension of Φ is a homeomorphism of $\overline{B}$ onto $\overline{D} \equiv \gamma \cup D$.

I will then present the main result. If $\Phi \in C^{1,\alpha}(\partial B)$, then we give a necessary and sufficient condition for U to be a diffeomorphism of $\overline{B}$ onto $\overline{D}$ so providing a sharp version of H. Kneser's Theorem. Finally, if time permits, I will present versions of Kneser's theorem which are valid when considering L^∞ elliptic operators rather than the Laplace operator with applications to composite materials. The talk is based upon a joint work with Giovanni Alessandrini, Università degli Studi di Trieste.